IATF Collins Rating Adjustments Calculation

The procedure for calculating rating adjustments is as follows:

Let the initial rating of a player be p_i and the initial rating of the player's opponent be q_i.

Let the player's match expectation value be E. The expectation value is calculated as:

$$E = 2 / \left(1 + 10^{\left(q_i - p_i\right) / 400}\right)$$

It follows that $p_i > q_i \Rightarrow 1 < E < 2$ and $p_i < q_i \Rightarrow 0 < E < 1$.

Each round in the match is evaluated to determine the result. A win is valued at 1, a loss at 0 and a tie at 0.5. In the case of a tie at 27, if $p_i > q_i$ the round is valued at half the expectation value ($E/2$).

The set of the player's round results is $\left\{g_1, g_2, \dots, g_n\right\}$ where g_1 is the player's result in round 1 and g_n is the player's result in round n and n is the number of rounds.

Let the player's match result value be R. The match result value is calculated as follows:

$R = \frac{2}{n} \sum_{k=1}^{n} g_k$, where n is the number of rounds.

Observe $\because 0 < g_k < 1 \Rightarrow 0 < R < 2$.

Let the player's rating adjustment be A. The adjustment is calculated as follows:

$A = \left[f^*(R - E)\right]$, where [ ] denotes rounding to the nearest integer, and f^* is a swing factor[1].

Let the player's final rating be p_f. The final rating is calculated as follows[2]:

$$p_f = p_i + A$$

Examples, for illustration:

Alice has a rating of 1700.
Bob has a rating of 1500.

Alice is the higher rated player, they are expected to win.

Alice's expectation value is $2 / \left(1 + 10^{(1500 - 1700) / 400}\right) \approx 1.52$.
Bob's expectation value is $2 / \left(1 + 10^{(1700 - 1500) / 400}\right) \approx 0.48$.

Alice's expectation value is higher than Bob's, since Alice is expected to win.

In round 1 Alice and Bob tie at 25.
In round 2 Alice wins 25-20.
In round 3 Alice wins 27-21.
Alice has won the match.

Alice's round results are: $\{0.5, 1, 1\}$.
Bob's round results are: $\{0.5, 0, 0\}$.

Alice's match result is: $\frac{2}{3} \times 2.5 \approx 1.67$.
Bob's match result is: $\frac{2}{3} \times 0.5 \approx 0.34$.

Alice's adjustment is: $10 \times (1.67 - 1.52) \approx + 1$ (here 10 is the swing factor).
Bob's adjustment is: $10 \times (0.34 - 0.48) \approx - 1$.

Alice's rating after the match is 1701.
Bob's rating after the match is 1499.

Since Alice won the match, and approximately by the expected amount, the adjustment is small.

Had the first round been a tie at 27 instead of 25, the results would follow:

Alice's round results are: $\{0.76, 1, 1\}$.
Bob's round results are: $\{0.5, 0, 0\}$.

Alice's match result is: $\frac{2}{3} \times 2.76 \approx 1.84$.
Bob's match result is: $\frac{2}{3} \times 0.5 \approx 0.34$.

Alice's adjustment is: $10 \times (1.84 - 1.52) \approx + 3$.
Bob's adjustment is: $10 \times (0.34 - 0.48) \approx - 1$.

Alice's rating after the match is 1703.
Bob's rating after the match is 1499.

Alice's performance is a little better and the adjustment is a little greater. The tie doesn't affect Bob differently though, a tie in this case is still better than their expectation.

Had Bob won rounds 2 & 3 (leaving the tie at 27), the results would follow:

Alice's round results are: $\{0.76, 0, 0\}$.
Bob's round results are: $\{0.5, 1, 1\}$.

Alice's match result is: $\frac{2}{3} \times 0.76 \approx 0.51$.
Bob's match result is: $\frac{2}{3} \times 2.5 \approx 1.67$.

Alice's adjustment is: $10 \times (0.51 - 1.52) \approx - 10$.
Bob's adjustment is: $10 \times (1.67 - 0.48) \approx + 12$.

Alice's rating after the match is 1690.
Bob's rating after the match is 1512.

Bob performed much better than expected and Alice performed worse. Their rating adjustments reflect the upset.

It should be noted that the first 28 matches a player completes in their career have an additional consideration, specifically, opponents' ratings in these 28 matches are not adjusted unless the match is also one of the opponents' first 28 matches. This prevents the uncertainty of new players' ratings from affecting their opponents' ratings in their first career season.

[1] The swing factor f^* is a parameter chosen to reflect the stakes of the type of match. The swing factor acts as a bounding factor on how large a rating adjustment can be for any given match of that type. Observe $|A| < 2f^*$. The swing factor generally increases with the level of competition or tournament profile. Regular league play has a swing factor of 10, larger tournaments can have swing factors of up to 20.

[2] The calculation can be expressed in terms of source input values as:

$$p_f = p_i + \left[2f^* \left(\frac{1}{n} \sum_{k=1}^{n} g_k - \left(1 + 10^{\left(q_i - p_i\right)/400} \right)^{-1} \right) \right]$$

Definitions

where

p_f is the player's final rating,

p_i is the player's initial rating,

q_i is the opponent's initial rating,

f^* is a swing factor,

g_k is the player's result in round k,

n is the number of rounds,

and $[\ \ ]$ denotes rounding to the nearest integer.